Threshold Resummation in Momentum Space from Effective Field Theory

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Fermilab theory seminar, 9/7/06

TB, M.Neubert, PRL 97, '06
TB, B. Pecjak and M. Neubert, hep-ph/0607228

Why resummation?

- Fixed order perturbation theory problematic for problems with widely separated scales $Q_1 >> Q_2$.
  - Large logarithms $\alpha_s^n \, \mathrm{Log}^n(Q_1/Q_2)$ and $\alpha_s^n \, \mathrm{Log}^{2n}(Q_1/Q_2)$. ← Sudakov logarithms
  - Scale in coupling? $\alpha_s(Q_1)$ or $\alpha_s(Q_2)$?
- Solution to both problems: integrate out physics at Q_1, solve RG, evolve to lower scale Q_2.
  - Effective theories

Resummation for collider processes

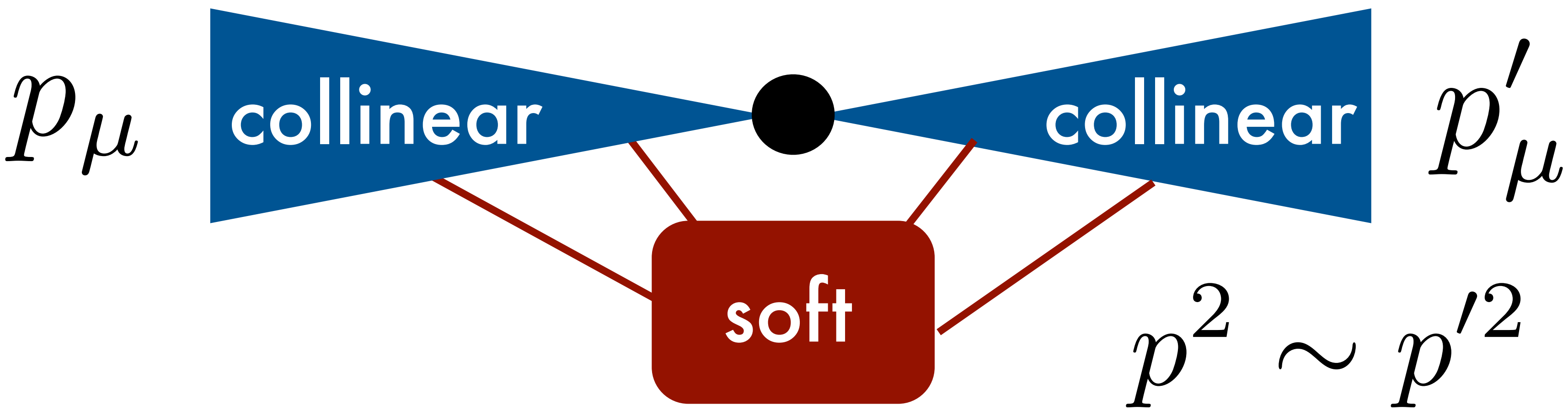
- An old problem! In the past 20 years resummations were performed for many processes with scale hierarchies
  - DIS for $x \to 1$, Drell-Yan and Higgs production for $Q^2/s \to 1$, for $Q_T^2/Q^2 \to 0$.
  - e^+e^- event shapes, hadronic event shapes, ...
  - ...
  - LL for arbitrary observable with parton shower
- Resummation is traditionally performed with diagrammatic methods.
  - No clear scale separation.

Bill Bardeen, in April:
“So why don’t you use your effective theory to do these resummations?”

Bill Bardeen, yesterday:
"Hadronic showers. Isn't it all Soft-Collinear Effective Theory?"

Soft-collinear effective theory

Bauer, Pirjol, Stewart '00



$$p^2 \sim p'^2 \ll 2p \cdot p'$$

- Eff. theory to analyze processes involving large momentum transfers and small invariant masses
- Originally developed to analyze B-meson decays to light hadrons
  - $B \to \pi\pi$, $B \to X_u l\nu$, ...

Work in  progress

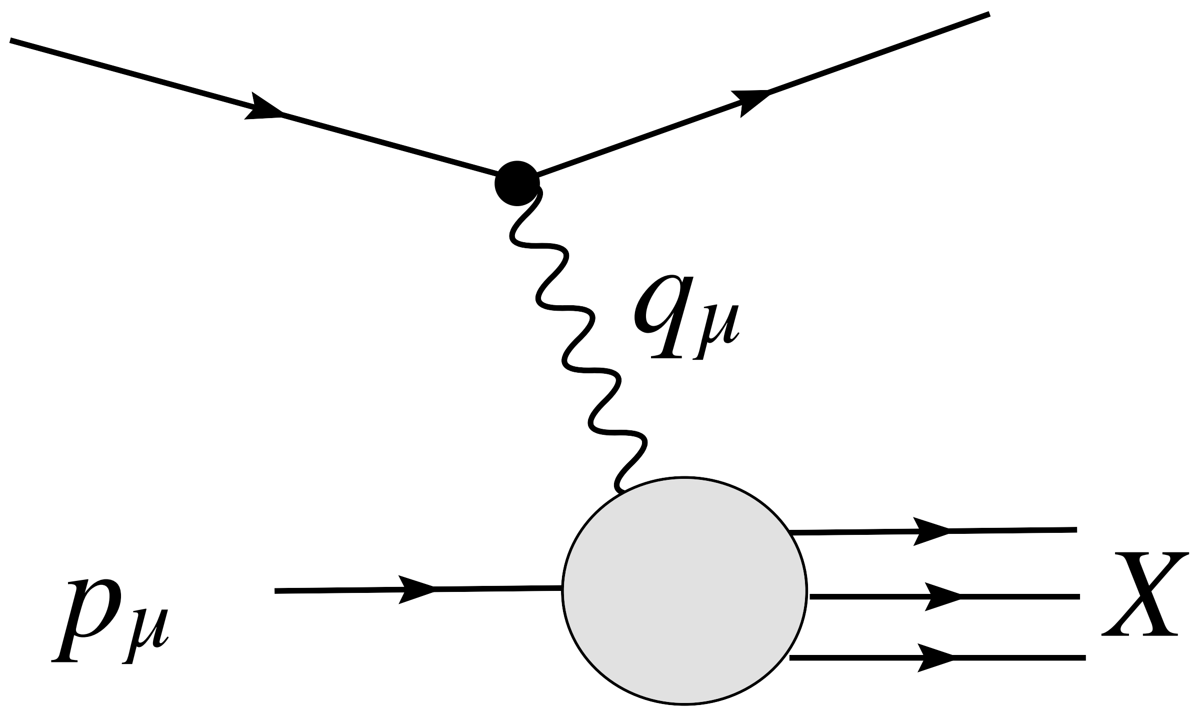
- So far, we have analyzed only simplest process, DIS for $x \to 1$ (as well as inclusive B-decays)
  - High precision: Next-to-next-to-next-to-leading logarithmic accuracy (N^3LL)
  - Detailed comparison with standard approach
  - Drell-Yan process and Higgs production for $Q^2/s \to 1$ underway. (See also Idilbi, Ji and Yuan, hep-ph/0605068.)
- Bauer and Schwartz: interesting proposal to improve parton showers with eff. theory
  - Not yet implemented, tested only at LL accuracy.

Outline

- DIS in the end-point region
- Factorization analysis
- Resummation
  - Traditional method
  - Using RG-evolution in SCET
  - Numerical results

Kinematics of DIS

$$e^-(k) + N(p) \to e^-(k') + X(P)$$



$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

- Are interested in the limit $x \to 1$, more precisely $Q^2 \gg Q^2(1-x) \gg \Lambda^2_{QCD}$

$$\approx M_X^2$$

Hadronic tensor

- Leptonic part factors off trivially

$$d\sigma = \frac{1}{2s}\frac{d^3k'}{k^0}\frac{1}{(q^2)^2}\,L^{\mu\nu}(k,q)W_{\mu\nu}(p,q)$$

$$L^{\mu\nu} = \frac{e^2}{8\pi^2}\mathrm{tr}\left[\not{k}\,\gamma^\mu\not{k}'\gamma^\mu\right] \qquad W_{\mu\nu} = \frac{1}{8\pi}\sum_\sigma\sum_X \langle N(p,\sigma)|\,J_\mu(0)\,|X\rangle\,\langle X|\,J_\nu(0)\,|N(p,\sigma)\rangle \times(2\pi)^4\delta(p_X-q-p)$$

- Optical theorem. Structure functions F_1 and F_2

$$W_{\mu\nu} = \frac{1}{8\pi}\,2\,\mathrm{Im}\,T_{\mu\nu} = \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)F_1(x,Q^2) + \left(\frac{p_\mu}{p\cdot q} - \frac{q_\mu}{q^2}\right)\left(\frac{p_\mu}{p\cdot q} - \frac{q_\mu}{q^2}\right)F_2(x,Q^2)$$

$$T_{\mu\nu} = i\int d^4x\,e^{iqx}\sum_\sigma \langle N(p,\sigma)|\,\mathrm{T}\left[J_\mu^\dagger(x)J_\nu(0)\right]\,|N(p,\sigma)\rangle$$

Factorization theorems

- Generic x

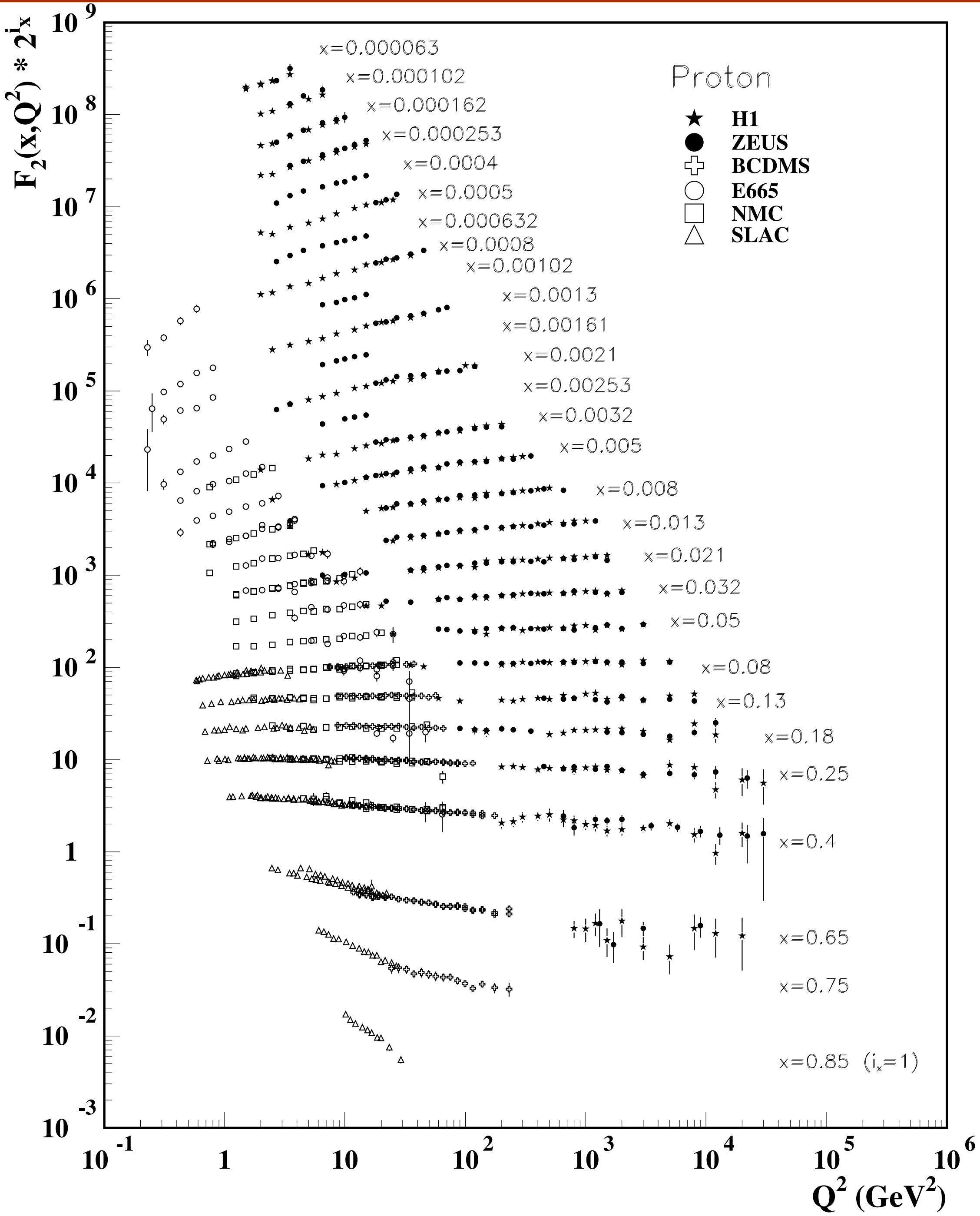
$$F_2^{\rm ns} = \int_x^1 \frac{dz}{z}\, \overbrace{C_2\left(z, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right)}^{\text{hard scattering coefficient}} \; \overbrace{\frac{x}{z}\phi_q^{\rm ns}(\frac{x}{z}, \mu^2)}^{\text{PDF}}$$

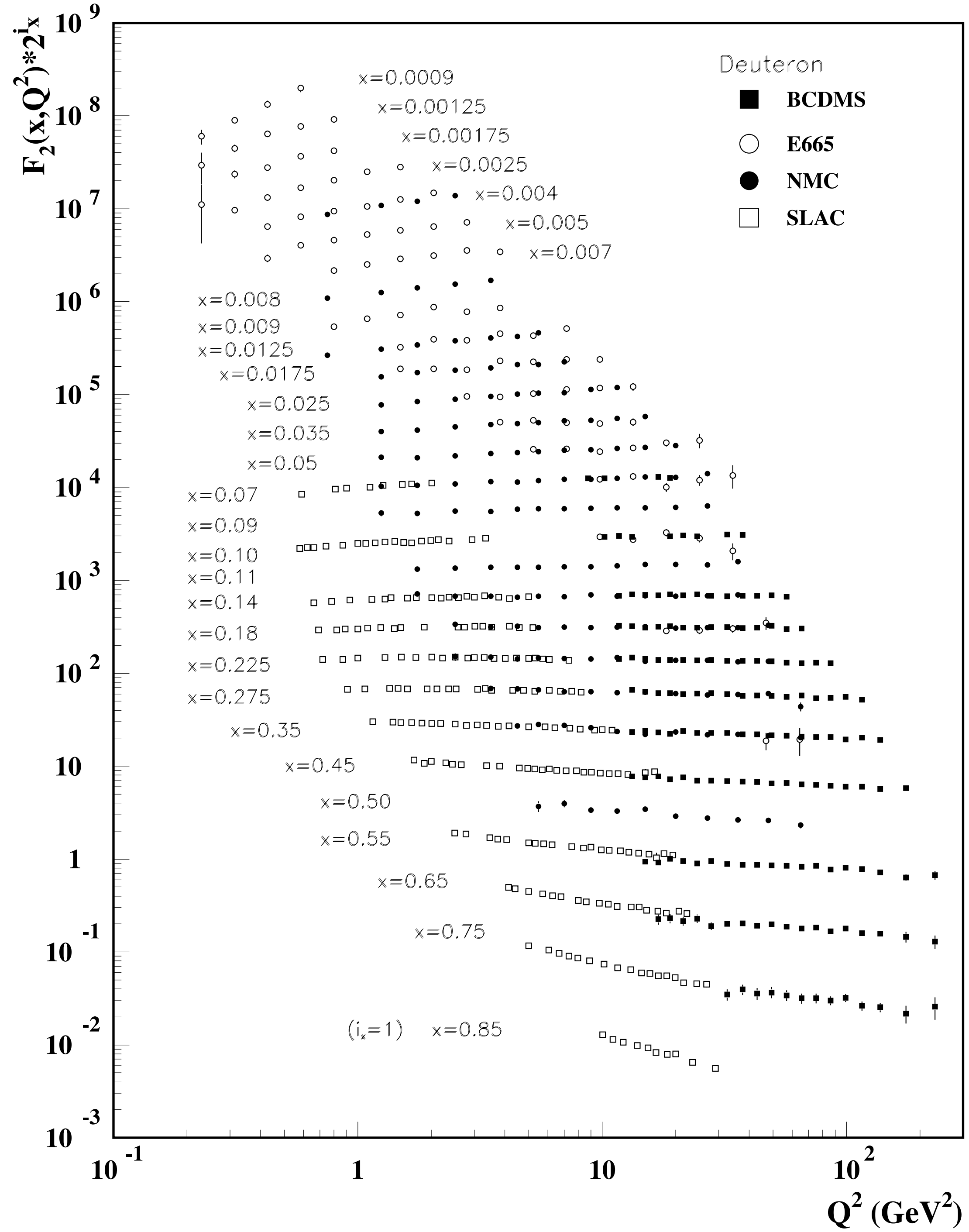
- End-point region $x \to 1$ $\;(Q^2 \gg M_X^2 \gg M_N^2)$

$$F_2^{\rm ns}(x, Q^2) = H(Q^2, \mu)\, Q^2 \int_x^1 \frac{dz}{z}\, J\Big(\underbrace{Q^2\,\frac{1-z}{z}}_{\approx M_X^2}, \mu\Big)\, \frac{x}{z}\, \phi_q^{\rm ns}\Big(\frac{x}{z}, \mu\Big)$$

Sterman '87

Measurements of F_2





Note: all measurements have $x \leq 0.85$.

Factorization analysis

310 7418240490 0437213507 5003588856 7930037346 0228427275 4572016194
8823206440 5180815045 5634682967 1723286782 4379162728 3803341547
1073108501 9195485290 0733772482 2783525742 3864540146 9173660247
7652346609

=

1634733 6458092538 4844313388 3865090859 8417836700 3309231218 1110852389
3331001045 0815121211 8167511579

x

1900871 2816648221 1312685157 3935413975 4718967899 6851549366 6638539088
0271038021 0449895719 1261465571

Factorization analysis in SCET

1. Start with QCD correlation function which describes process under consideration.
2. Identify relevant momentum regions for its expansion around $Q^2 \to \infty$
3. Introduce corresponding effective theory fields. Derive Lagrangian.
4. Identify operator basis
    1. Matching coefficients: partonic hard scattering amplitudes
    2. Matrix elements: PDFs

Analysis is technical. Wear lab coat for protection!

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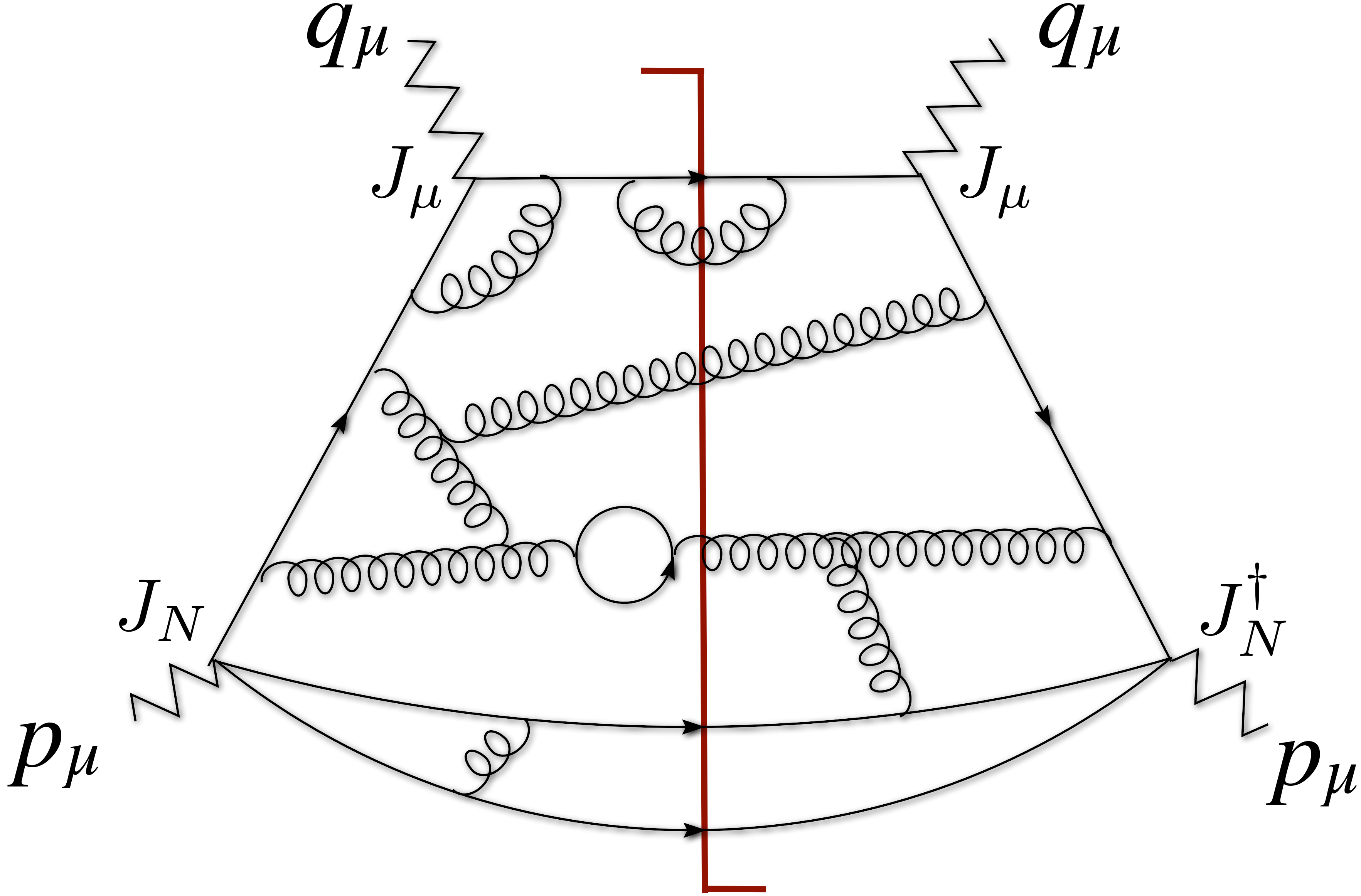
Earlier work in SCET (w/o lab coats)

- Previous analyses found:
  - ☹ Different number of momentum regions in different frames.
  - ☹ Non-factorization as $x \to 1$.
  - ☹ Factorization, but additional soft contributions which are not part of the PDF.
  - ☹ Ignore perturbative non-factorization $\to$ Non-perturbative factorization due to confinement.

Correlator (whale) diagrams



- Now expand around limit $Q^2 \gg Q^2(1-x) \gg p^2$

Momentum regions

- Light-cone components $(n \cdot k, \bar{n} \cdot k, k_\perp^\mu)$ in Breit frame:

$$k^2 = n \cdot k\, \bar{n} \cdot k + k_\perp^2$$

hard-collinear:
$p_X^\mu \sim Q(\epsilon, 1, \sqrt{\epsilon})$

$$\epsilon = 1 - x$$

$$\lambda \sim m_N/Q \sim \Lambda_{\mathrm{QCD}}/Q$$

hard:
$Q(1, 1, 1)$

anti-collinear:
$p^\mu \sim Q(1, \lambda^2, \lambda)$

soft-collinear:
$Q(\epsilon, \lambda^2, \sqrt{\epsilon}\lambda)$

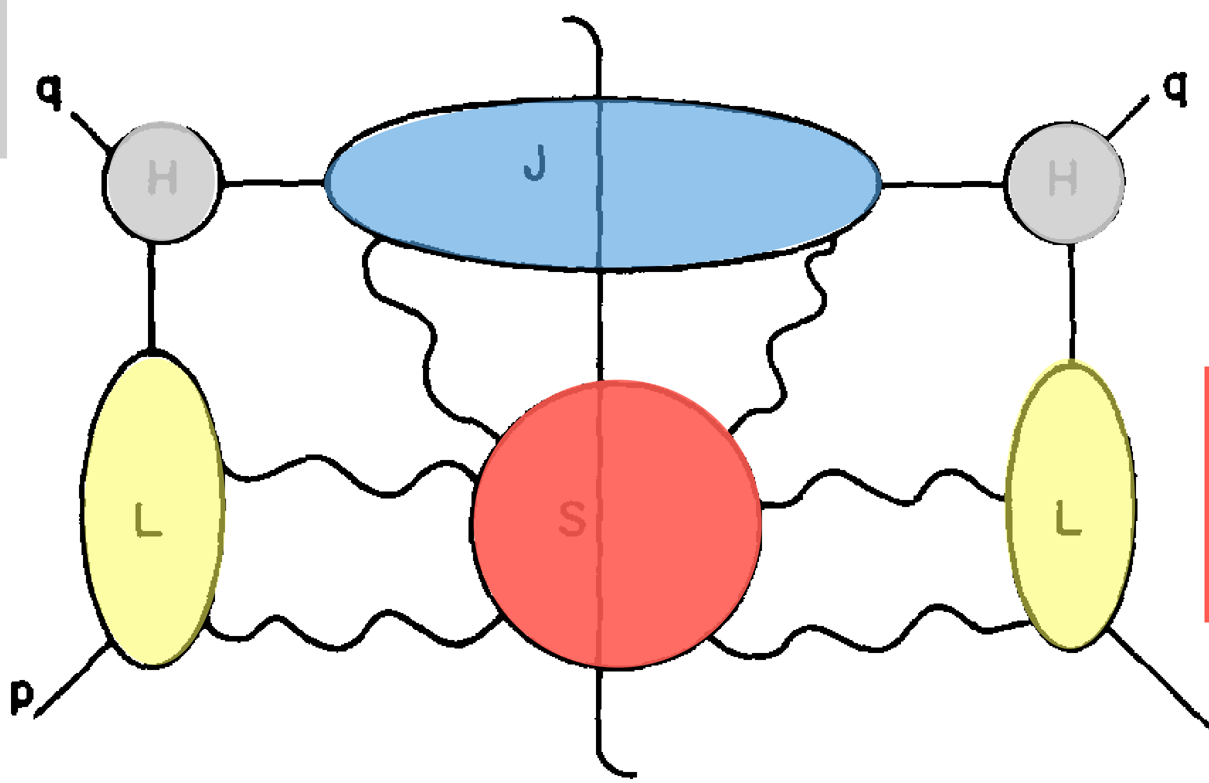
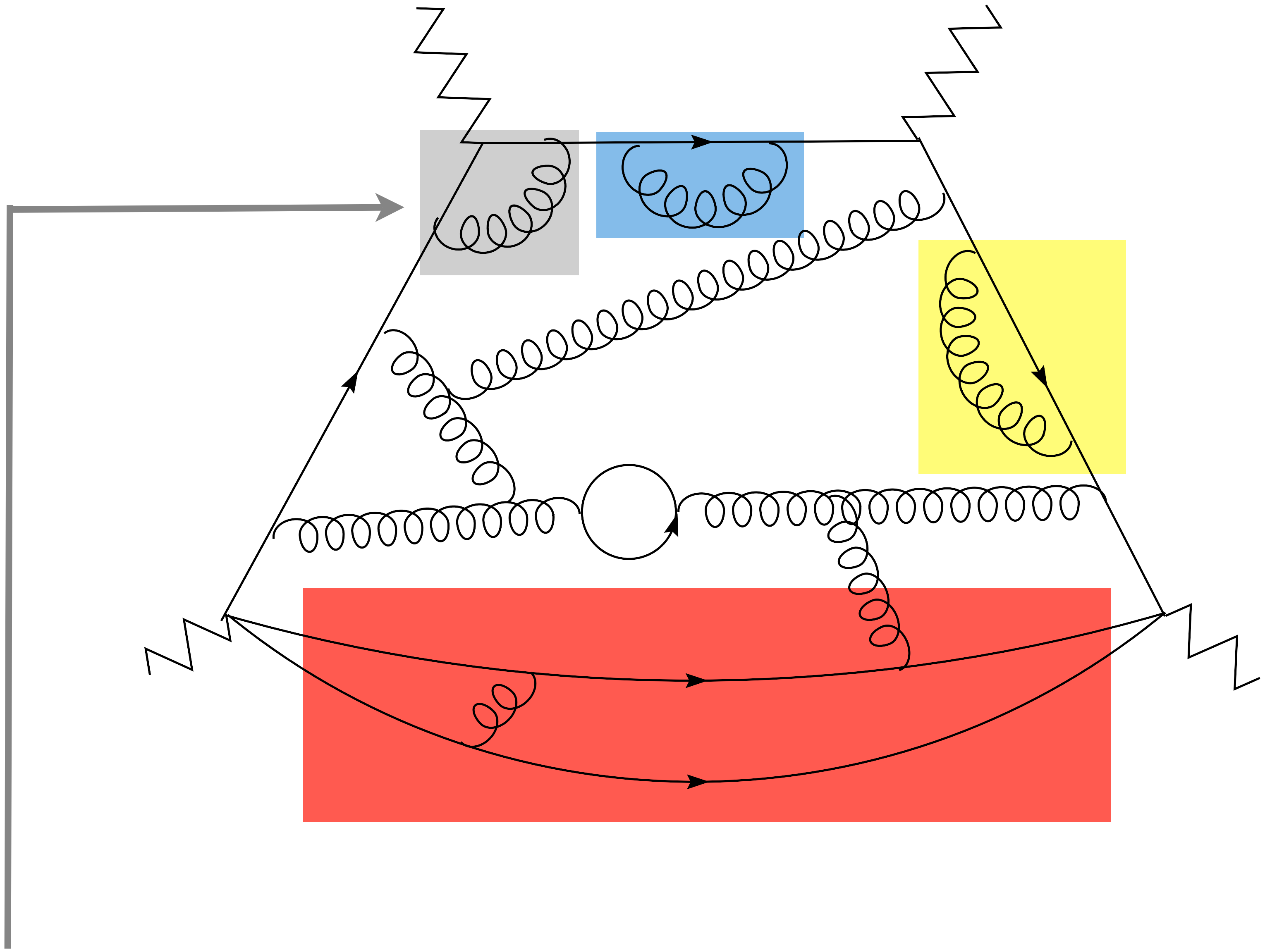


Fig. 3 1. Leading regions for DIS.

Sterman '87

Example diagram



- Note: given loop in general has contributions from several momentum regions.

Effective theory

- Lagrangian
  - Two components of collinear quark fields ξ_{hc} and ξ_c are integrated out. Derivative expansion of $\mathcal{L}$.

$$\mathcal{L}_{\rm SCET}(y) = \bar{\xi}_{hc}\,\frac{\not{\bar{n}}}{2}\,[in\cdot D_{hc} + gn\cdot A_{sc}(y_-)]\,\xi_{hc} - \bar{\xi}_{hc}\,i\not{D}_{hc\perp}\,\frac{\not{\bar{n}}}{2}\,\frac{1}{i\bar{n}\cdot D_{hc}}\,i\not{D}_{hc\perp}\,\xi_{hc}$$

$$+\,\bar{\xi}_{\bar{c}}\,\frac{\not{n}}{2}\,[i\bar{n}\cdot D_{\bar{c}} + g\bar{n}\cdot A_{sc}(y_+)]\,\xi_{\bar{c}} - \bar{\xi}_{\bar{c}}\,i\not{D}_{\bar{c}\perp}\,\frac{\not{n}}{2}\,\frac{1}{in\cdot D_{\bar{c}}}\,i\not{D}_{\bar{c}\perp}\,\xi_{\bar{c}}$$

- Current operator $W_{hc}(x) = \mathbf{P}\,\exp\left(ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A_{hc}(x+s\bar{n})\right)$

$$(\bar{\psi}\gamma^\mu\psi)(x) \to \int dt\,\widetilde{C}_V(t,n\cdot q,\mu)\,(\bar{\xi}_{\bar{c}}W_{\bar{c}})(x_-)\,\gamma_\perp^\mu(W_{hc}^\dagger\xi_{hc})(x+t\bar{n})$$

Current matching

- Bare Wilson coefficient C_V is on-shell QCD form factor.

  2-loop result: Matsuura and van Neerven '89

  - eff. theory loop diagrams vanish on shell (because they are scaleless).

- UV divergencies in eff. theory are equal IR to divergencies in QCD.

  3-loop div's: Moch, Vermaseren and Vogt '05

$$C_V(Q^2,\mu) = \lim_{\epsilon\longrightarrow 0} Z_V^{-1}(\epsilon, Q^2, \mu)\, F_{\text{bare}}(\epsilon, Q^2)$$

Wilson coefficient C_V

- 2-loop result: L=ln(Q^2/μ^2)

$$C_V(Q^2,\mu) = 1+\frac{C_F\alpha_s}{4\pi}\left(-L^2+3L-8+\frac{\pi^2}{6}\right)+C_F\left(\frac{\alpha_s}{4\pi}\right)^2\left[C_F H_F + C_A H_A + T_F n_f H_f\right],$$

with

$$\begin{aligned}
H_F &= \frac{L^4}{2} - 3L^3 + \left(\frac{25}{2}-\frac{\pi^2}{6}\right)L^2 + \left(-\frac{45}{2}-\frac{3\pi^2}{2}+24\zeta_3\right)L + \frac{255}{8} + \frac{7\pi^2}{2} - \frac{83\pi^4}{360} - 30\zeta_3\,,\\
H_A &= \frac{11}{9}L^3 + \left(-\frac{233}{18}+\frac{\pi^2}{3}\right)L^2 + \left(\frac{2545}{54}+\frac{11\pi^2}{9}-26\zeta_3\right)L\\
&\quad - \frac{51157}{648} - \frac{337\pi^2}{108} + \frac{11\pi^4}{45} + \frac{313}{9}\zeta_3\,,\\
H_f &= -\frac{4}{9}L^3 + \frac{38}{9}L^2 + \left(-\frac{418}{27}-\frac{4\pi^2}{9}\right)L + \frac{4085}{162} + \frac{23\pi^2}{27} + \frac{4}{9}\zeta_3\,.
\end{aligned} \qquad (51$$

Decoupling transformation

$$T_{\mu\nu} = i\int d^4x\, e^{iqx} \sum_\sigma \langle N(p,\sigma)|\,\mathrm{T}\left[J_\mu^\dagger(x) J_\nu(0)\right]\,|N(p,\sigma)\rangle$$
$$= |C_V(Q^2,\mu)|^2\, i\int d^4x\, e^{iq\cdot x} \langle N(p)|\, T\{(\bar\xi_{\bar c} W_{\bar c})(x_-)\,\gamma_\perp^\mu\, (W_{hc}^\dagger \xi_{hc})(x) (\bar\xi_{hc} W_{hc})(0)\, \gamma_\perp^\nu\, (W_{\bar c}^\dagger \xi_{\bar c})(0)\}\,|N(p)\rangle$$

- Redefine

$$\xi_{hc}(x) \to S_n(x_-)\,\xi_{hc}^{(0)}(x)\,, \qquad A_{hc}^\mu(x) \to S_n(x_-)\,A_{hc}^{\mu(0)}(x)\,S_n^\dagger(x_-)$$

$$S_n(x) = \mathbf{P}\,\exp\left(ig\int_{-\infty}^0 ds\, n\cdot A_{sc}(x+sn)\right) \qquad S_n^\dagger(in\cdot\partial + n\cdot A_{sc})S_n = in\cdot\partial$$

  - The new hc-fields no longer interact with sc and c.

- Hard-collinear matrix element is jet function:

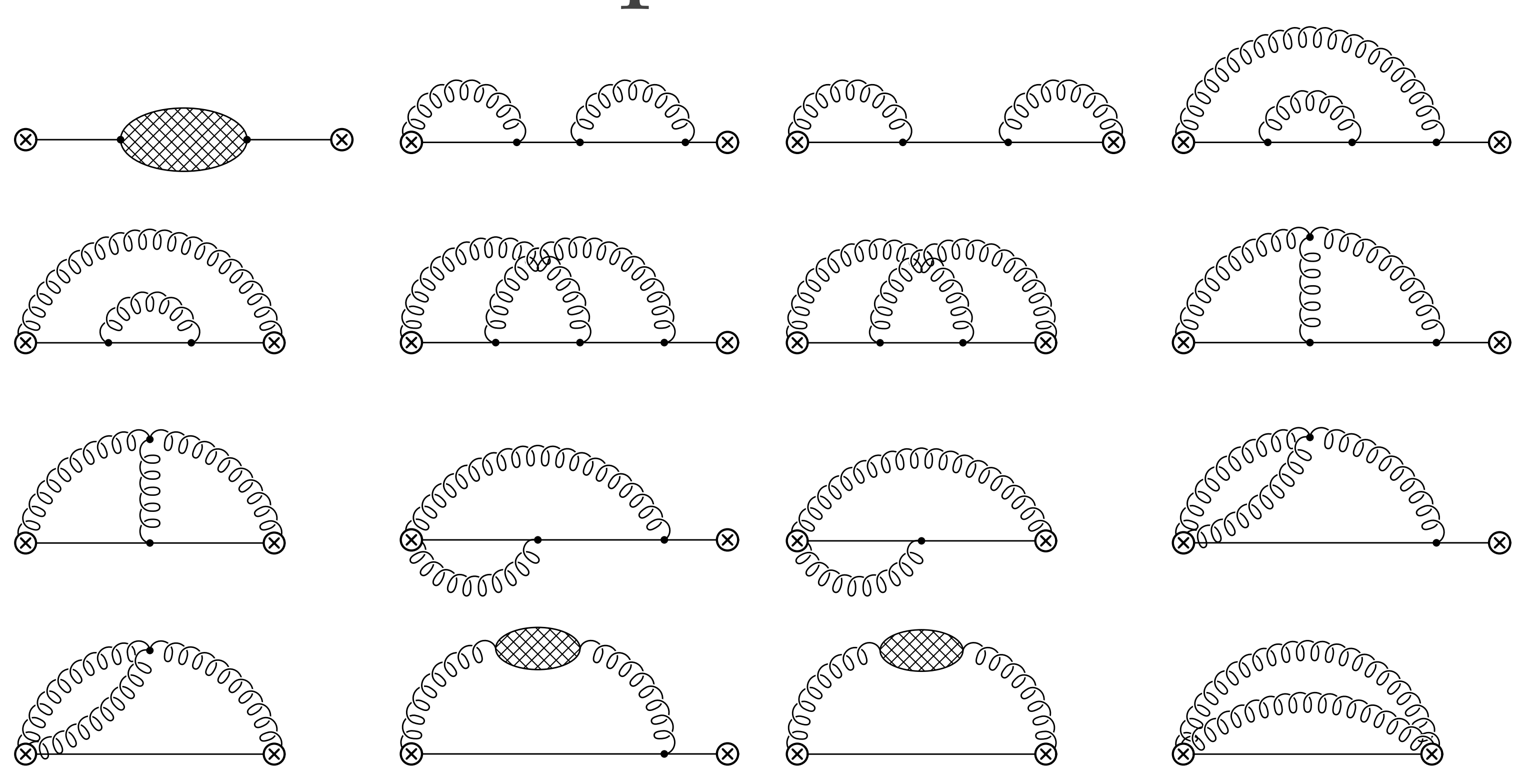
$$\langle 0|\, T\{(W_{hc}^{(0)\dagger}\,\xi_{hc}^{(0)})(x)\;(\bar\xi_{hc}^{(0)}\,W_{hc}^{(0)})(0)\}\,|0\rangle = \int \frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x}\, \frac{\not n}{2}\, \bar n\cdot k\, \mathcal{J}(k^2,\mu)$$

Jet function

- Can rewrite jet-function in terms of QCD fields

$$\frac{\not{n}}{2}\,\bar{n}\cdot p\,\mathcal{J}(p^2,\mu) = \int d^4x\, e^{-ip\cdot x}\,\langle 0\,|\,\mathrm{T}\left\{\frac{\not{n}\bar{\not{n}}}{4}\,W^\dagger(0)\,\psi(0)\,\overline{\psi}(x)\,W(x)\,\frac{\bar{\not{n}}\not{n}}{4}\right\}|\,0\rangle$$

- Known to 2 loops.



TB, M. Neubert '06

Parton distribution function

$$\langle N(p)|\,(\bar{\xi}_{\bar{c}}W_{\bar{c}})(x_-)\,S_n(x_-)\,\gamma_\perp^\mu\,\frac{\not{n}}{2}\,\gamma_\perp^\nu\,S_n^\dagger(0)\,(W_{\bar{c}}^\dagger\xi_{\bar{c}})(0)\,|N(p)\rangle$$

$$= -\langle N(p)|\,(\bar{\xi}_{\bar{c}}W_{\bar{c}})(x_-)\,[x_-,0]_{sc}\,(g_\perp^{\mu\nu} - i\epsilon_\perp^{\mu\nu}\gamma_5)\,\frac{\not{n}}{2}\,(W_{\bar{c}}^\dagger\,\xi_{\bar{c}})(0)\,|N(p)\rangle$$

→ Callen-Gross relation

vanishes when averaged over spin

- Remaining matrix element is PDF in the end-point

$$\phi_q^{\rm ns}(\xi,\mu)|_{\xi\to 1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i\xi t n\cdot p}\,\langle N(p)|\,(\bar{\xi}_{\bar{c}}W_{\bar{c}})(tn)\,[tn,0]_{sc}\,\frac{\not{n}}{2}\,(W_{\bar{c}}^\dagger\xi_{\bar{c}})(0)\,|N(p)\rangle$$

- Factorization theorem

$$F_2^{\rm ns}(x,Q^2) = \sum_q e_q^2\,|C_V(Q^2,\mu)|^2\,Q^2\int_x^1 d\xi\, J\Big(Q^2\,\frac{\xi-x}{x},\mu\Big)\,\phi_q^{\rm ns}(\xi,\mu)$$

Cusp singularities

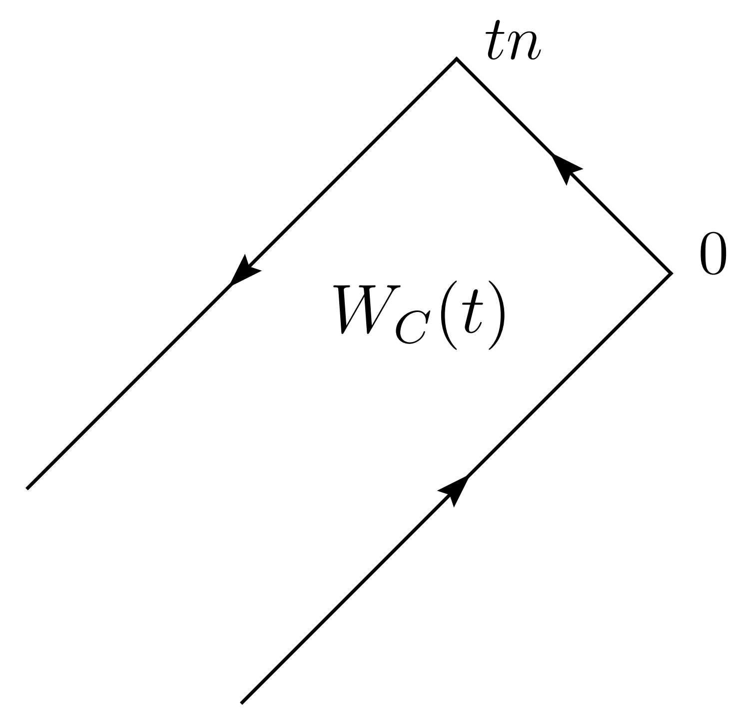
- Decouple *sc* from collinear fields:

$$\xi_{\bar c}(x_-) \to S_{\bar n}(x_-)\, \xi_{\bar c}^{(0)}(x_-)\,, \qquad A_{\bar c}^{\mu}(x_-) \to S_{\bar n}(x_-)\, A_{\bar c}^{\mu(0)}(x_-)\, S_{\bar n}^{\dagger}(x_-)$$

- PDF factorizes

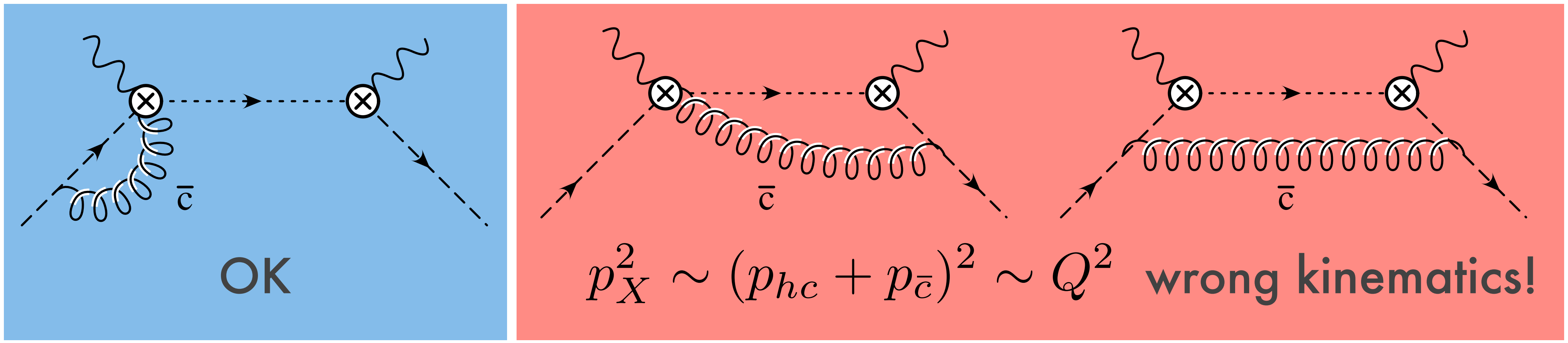
$$\phi_q^{\rm ns}(\xi,\mu)|_{\xi\to 1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i\xi t n\cdot p}\, \langle N(p)|\, (\bar\xi_{\bar c}^{(0)} W_{\bar c}^{(0)})(tn)\, \frac{\not n}{2}\, W_C(t)\, (W_{\bar c}^{(0)\dagger} \xi_{\bar c}^{(0)})(0)\, |N(p)\rangle$$

$$\boxed{W_C(t) = S_{\bar n}^{\dagger}(tn)\, [tn, 0]_{sc}\, S_{\bar n}(0)}$$

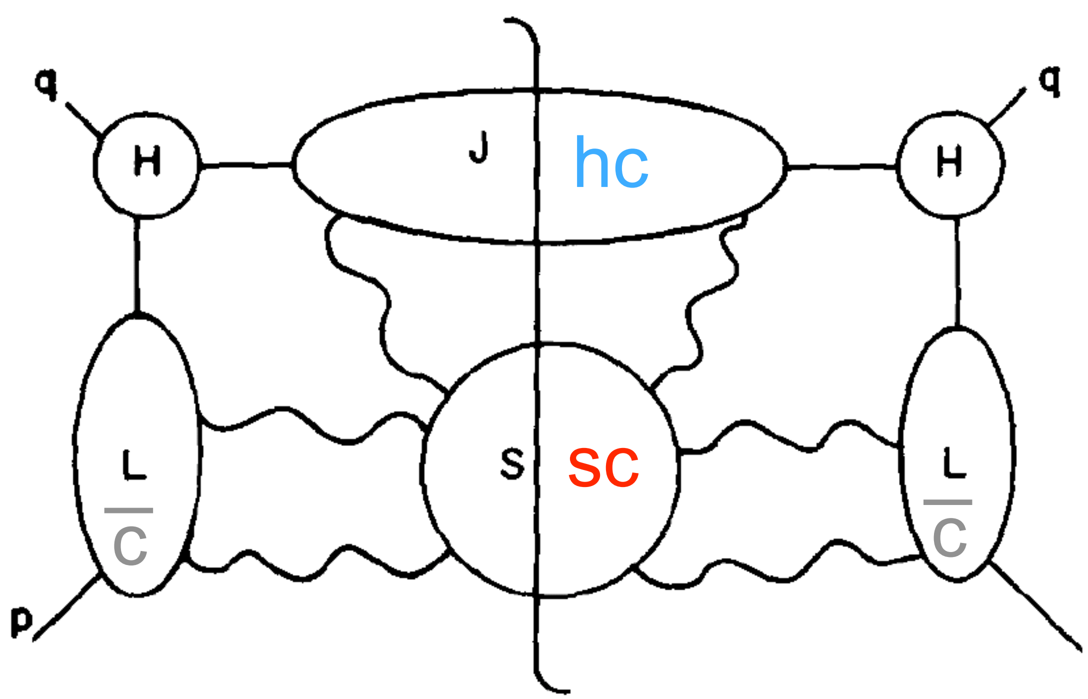


- Explains occurence cusp anomalous dimension in Altarelli-Parisi kernel.
- Both contributions non-perturbative.

A subtlety



- The two anti-collinear fields can only interact via *sc*-exchange, not directly!



Factorization summary

$$F_2^{\mathrm{ns}}(x,Q^2) = \sum_q e_q^2 \, |C_V(Q^2,\mu)|^2 \, Q^2 \int_x^1 d\xi \, J\Big(Q^2 \frac{\xi - x}{x}, \mu\Big) \, \phi_q^{\mathrm{ns}}(\xi,\mu)$$

hard part OS form factor	hard-collinear propagator in LC gauge	anti-collinear + soft-collinear PDF for $\xi \to 1$

- Any choice of the scale μ will lead to large perturbative logarithms.
  - Solve RG for individual parts, evolve to common scale.

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Transverse momentum **resummation** demo
Transverse momentum **resummation** demo. ... **Resummation** program : C. Balazs, G. Ladinsky, C.-P. Yuan (Fortran); P. Nadolsky (C++); CTEQ Collaboration (CTEQ ...

Traditional method: moment space

Sterman '87, Catani and Trentadue '89

$$F^{\rm ns}_{2,N}(Q^2) = \int_0^1 dx\, x^{N-1} F_2^{\rm ns}(x,Q^2)$$
$$= C_N(Q^2,\mu_f) \sum_q e_q^2\, \phi^{\rm ns}_{q,N}(\mu_f)$$

- Convolution in momentum space $\rightarrow$ product in moment space
- $x\rightarrow 1$ corresponds to $N\rightarrow\infty$. Perturbation theory contains $\alpha_s^n \mathrm{Log}^n(N)$ and $\alpha_s^n \mathrm{Log}^{2n}(N)$
- Split:

$$C_N(Q^2,\mu_f) = g_0(Q^2,\mu_f)\, \exp\left[G_N(Q^2,\mu_f)\right]$$

Resummation in moment space

$$C_N(Q^2, \mu_f) = g_0(Q^2, \mu_f)\, \exp\left[G_N(Q^2, \mu_f)\right]$$

$$G_N(Q^2, \mu_f) = \int_0^1 dz\, \frac{z^{N-1} - 1}{1 - z}$$

$$\times \left[\int_{\mu_f^2}^{(1-z)Q^2} \frac{dk^2}{k^2} A_q(\alpha_s(k)) + B_q\left(\alpha_s(Q\sqrt{1-z})\right)\right]$$

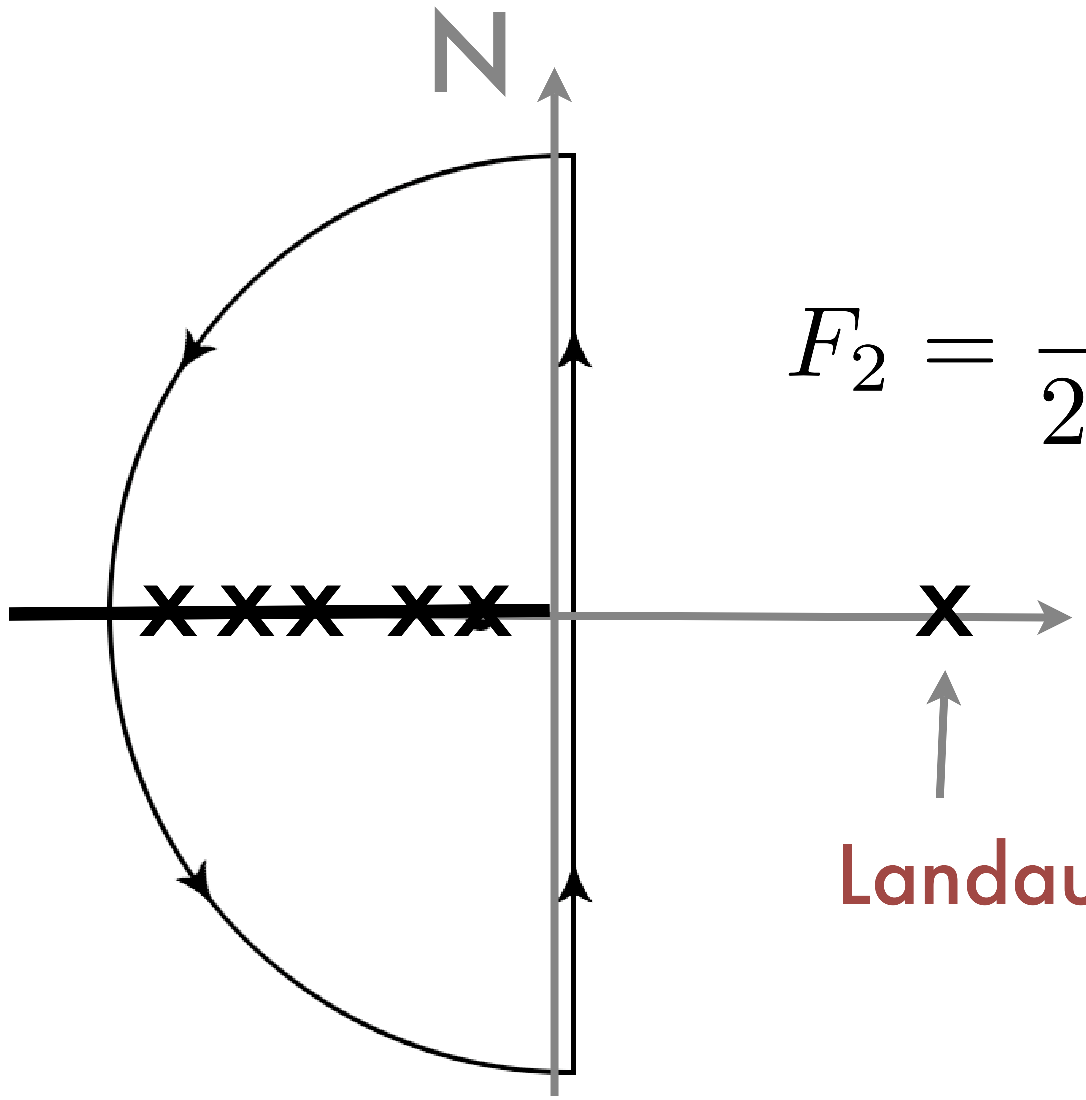
Landau pole

Cusp anomalous dim.

Anom. dim. of ??

- A_q, B_q determined by matching to fixed order result. NNNLL: Moch, Vermaseren, Vogt '05

Mellin Inversion



$$F_2 = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\phi_N\, C_N$$

- Can only be done numerically
- Problem with Fortran PDF's.

Resummation by RG evolution: 1. hard part

- RG equation for C_V

$$\frac{d}{d\ln\mu} C_V(Q^2,\mu) = \left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + \gamma^V(\alpha_s)\right] C_V(Q^2,\mu)$$

structure: TB, Hill, Lange, Neubert '03
3-loop anomalous dim.: Moch, Vermaseren Vogt '05

- Solution

$$C_V(Q^2,\mu) = \exp\left[2S(\mu_h,\mu) - a_{\gamma^V}(\mu_h,\mu)\right]\left(\frac{Q^2}{\mu_h^2}\right)^{-a_\Gamma(\mu_h,\mu)} C_V(Q^2,\mu_h)$$

$$S(\nu,\mu) = -\int\limits_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}\int\limits_{\alpha_s(\nu)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')}\,, \qquad a_\Gamma(\nu,\mu) = -\int\limits_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\,\frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

Resummation by RG evolution: 2. jet function

$$\frac{dJ(p^2,\mu)}{d\ln\mu} = -\left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{p^2}{\mu^2} + 2\gamma^J(\alpha_s)\right] J(p^2,\mu) - 2\Gamma_{\rm cusp}(\alpha_s)\int_0^{p^2} dp'^2\,\frac{J(p'^2,\mu)-J(p^2,\mu)}{p^2-p'^2}$$

$$J(p^2,\mu) = \exp\left[-4S(\mu_i,\mu) + 2a_{\gamma^J}(\mu_i,\mu)\right] \times \widetilde{j}(\partial_\eta,\mu_i)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\,\frac{1}{p^2}\left(\frac{p^2}{\mu_i^2}\right)^\eta,$$

$$\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_{\rm cusp}[\alpha_s(\mu)] = 2a_\Gamma(\mu_i,\mu).$$

- Associated jet-function $\widetilde{j}$ is Laplace transform of $J(p^2,\mu_i)$.

$$\widetilde{j}\left(\ln\frac{Q^2}{\mu^2},\mu\right) = \int_0^\infty dp^2\,e^{-sp^2}\,J(p^2,\mu) \quad \text{with} \quad s = 1/(e^{\gamma_E}Q^2)$$

Resummation by RG evolution: 3. PDF near the end-point

$$\frac{d}{d\ln\mu}\,\phi_q^{\rm ns}(\xi,\mu) \;=\; \int_\xi^1 \frac{dz}{z}\,P_{q\leftarrow q}^{(\rm endpt)}(z)\,\phi_q^{\rm ns}\Big(\frac{\xi}{z},\mu\Big)$$

$$P_{q\leftarrow q}^{(\rm endpt)}(z) = \frac{2\Gamma_{\rm cusp}(\alpha_s)}{(1-z)_+} + 2\gamma^\phi(\alpha_s)\,\delta(1-z)$$

- Equation (and its solution) can be obtained from

$$\frac{d}{d\ln\mu}F_2(x,Q^2) = 0$$

- Can obtain 3-loop γ^J using

$$\gamma^J = \gamma^\phi + \gamma^V$$

Moch, Vermaseren Vogt '04

Result for F_2

- Evolve C_V and J from μ_h and μ_i to scale μ_f, plug into factorization theorem

$$F_2^{\rm ns}(x,Q^2) = \sum_q e_q^2\, |C_V(Q^2,\mu_h)|^2\, U(Q,\mu_h,\mu_i,\mu_f)$$
$$\times \widetilde{j}\Big(\ln\frac{Q^2}{\mu_i^2}+\partial_\eta,\mu_i\Big)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\int_x^1 d\xi\,\frac{\phi_q^{\rm ns}(\xi,\mu_f)}{(\xi-x)^{1-\eta}}$$

$$U(Q,\mu_h,\mu_i,\mu_f) = \exp\left[4S(\mu_h,\mu_i) - 2a_{\gamma_V}(\mu_h,\mu_i)\right]$$
$$\times\left(\frac{Q^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} \exp\left[2a_{\gamma^\phi}(\mu_i,\mu_f)\right]\,,$$

Result

- If we assume $\phi_q(x,\mu_f) \sim (1\text{-}x)^{b(\mu f)}$:

$$\frac{F_2^{\rm ns}(x,Q^2)}{\sum_q e_q^2\,x\,\phi_q^{\rm ns}(x,\mu_f)} = |C_V(Q^2,\mu_h)|^2\,U(Q,\mu_h,\mu_i,\mu_f) \times (1-x)^\eta\,\widetilde{j}\Big(\ln\frac{Q^2(1-x)}{\mu_i^2}+\partial_\eta,\mu_i\Big) \times \frac{e^{-\gamma_E\eta}\,\Gamma(1+b(\mu_f))}{\Gamma(1+b(\mu_f)+\eta)}\,.$$

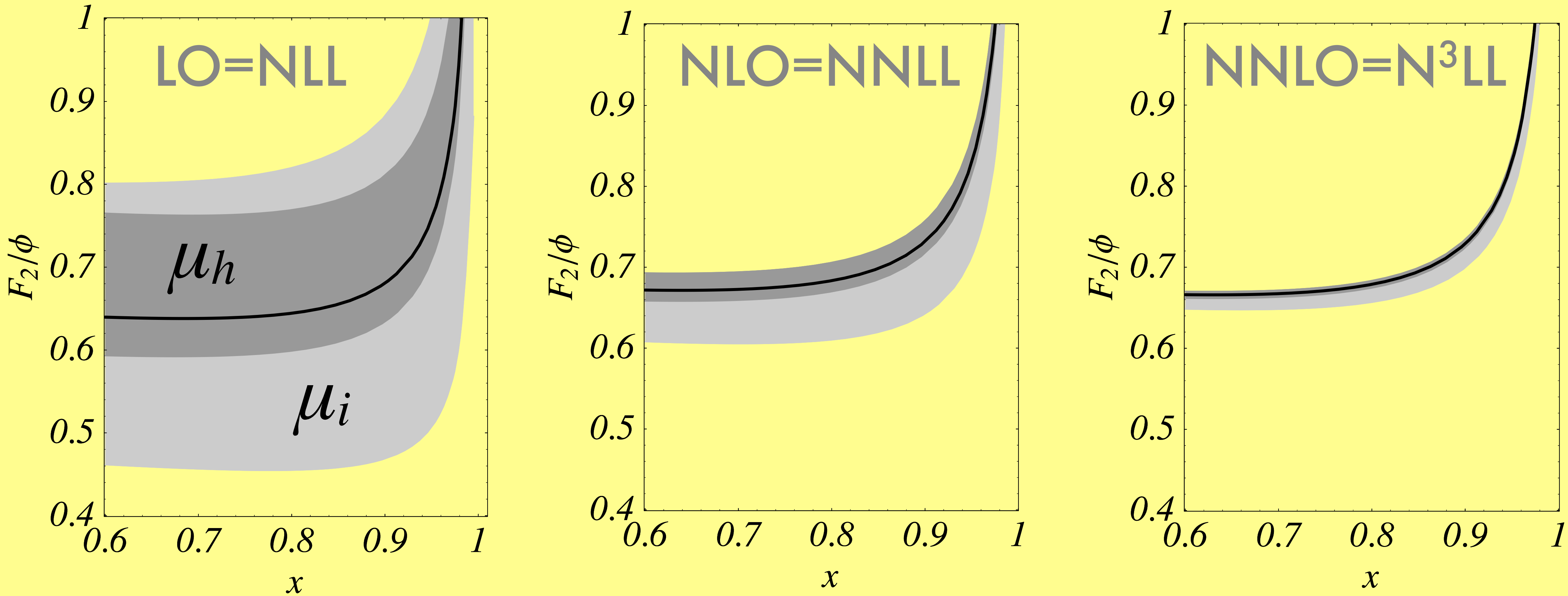
- Resummed result obtained after plugging in fixed order results for coefficient C_V, jet-function and anom. dimensions.

Difference to traditional approach

- Simple analytic result in momentum space
- No Landau pole ambiguities. No coupling constant below scales μ_h, μ_i and μ_f.
- Freedom to choose scales μ_h, μ_i and μ_f
  - Obtain fixed order for μ_h=μ_i=μ_f. Trivial matching to fixed order result for generic x.
  - Set appropriate scales *after* integrating
    - Avoids large spurious power corrections discussed by Catani et al. hep-ph/9604351
  - Estimate uncertainties with scale variation

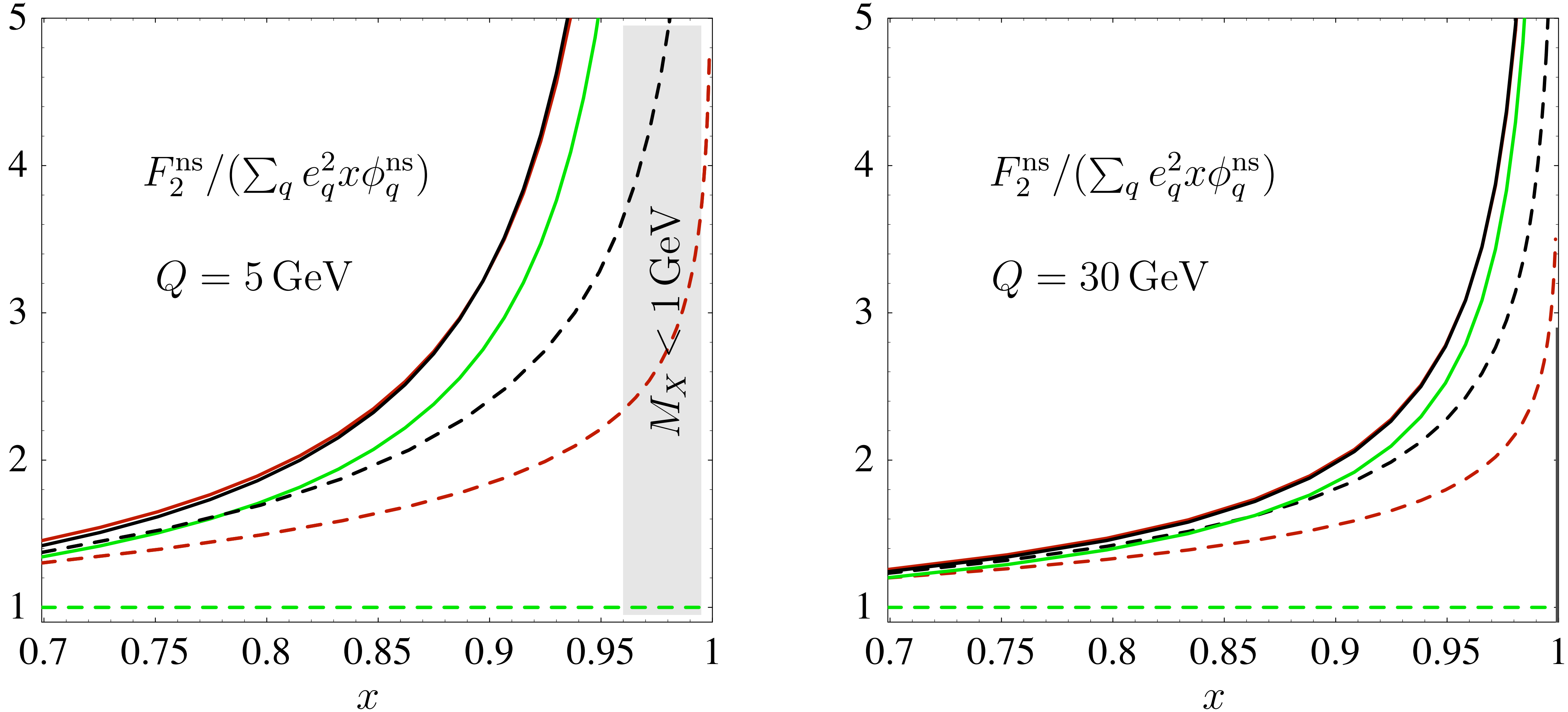
Result for $F_2^{\mathrm{ns}}(\mathrm{x})/\phi_q(x)$

$$Q = 30\,\mathrm{GeV}, \quad \mu_f = 5\mathrm{GeV}, \quad \phi(x,\mu_f) \sim (1-x)^4$$



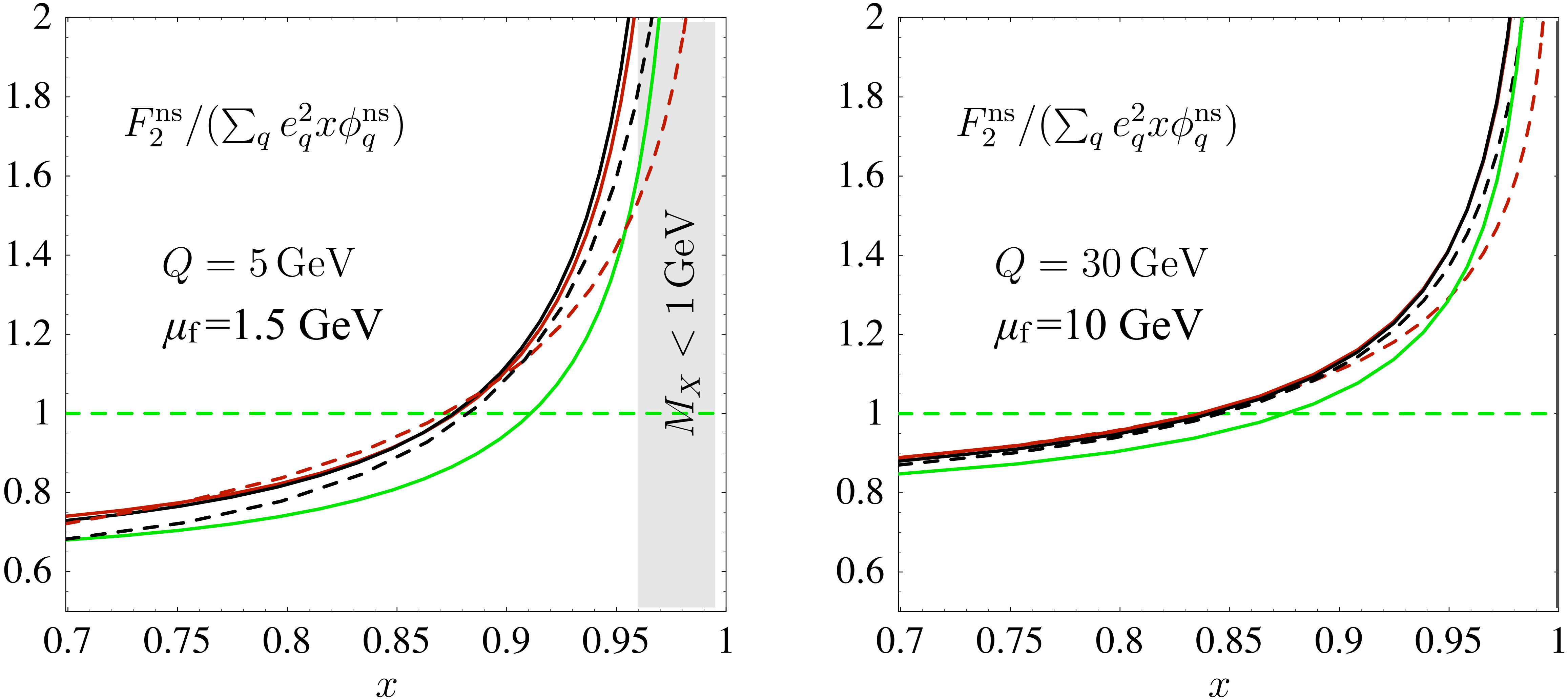
- Default scales: $\mu_h^2 = Q^2$ and $\mu_i^2 = Q^2(1\text{-x})$
  - Bands obtained by varying these scales a factor of two up and down.
  - Matching scales are fixed in traditional approach.

Comparison with fixed order, $\mu_f = Q$



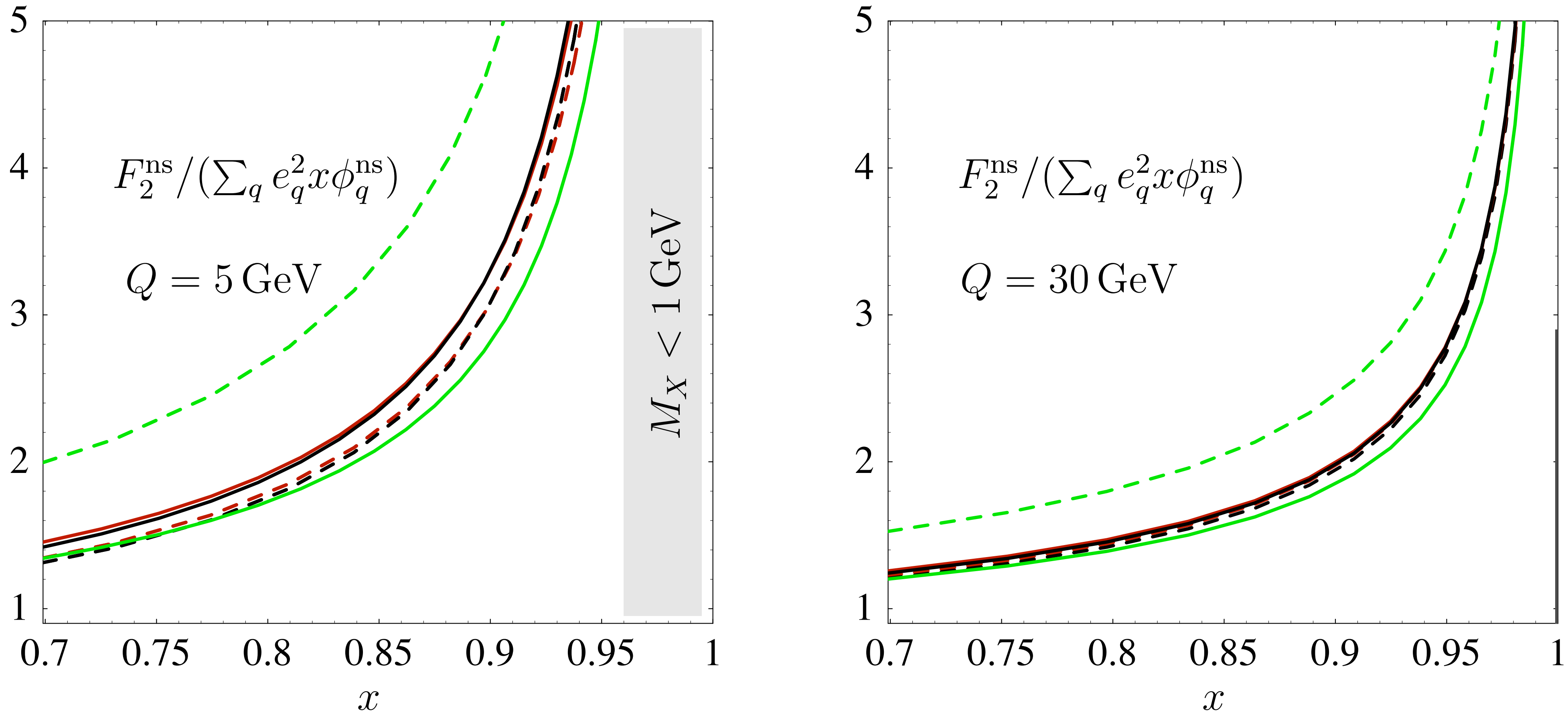
- LO (=NLL), NLO, NNLO
- Dashed: fixed order. Solid: resummed.
- Large K-factors.

Comparison with fixed order, low μ_f



- LO (=NLL), NLO, NNLO
- Dashed: fixed order. Solid: resummed.
- Fixed order with $\mu=\mu_{\rm f}$ fairly close to resummed result!

Comparison with moment space result



- Dashed: Mellin inverted moment space results. Solid: momentum space results.
- Only small numerical differences (different scale choice, 1/N corrections in moment space).
- Faster convergence of momentum space results.

Connection with standard approach

- Can compare EFT expression for moments with standard results. The two agree provided that

$$\left(1+\frac{\pi^2}{12}\nabla^2+\dots\right)B_q(\alpha_s)=\gamma^J(\alpha_s)+\nabla\,\ln\widetilde{j}(0,\mu)$$
$$-\left(\frac{\pi^2}{12}\nabla-\frac{\zeta_3}{3}\nabla^2+\dots\right)\Gamma_{\text{cusp}}(\alpha_s)\,,\qquad \nabla\;=\;d/d\ln\mu^2.$$

- fulfilled with two-result from explicit calculation of $J(p^2)$.

Momentum space?

- Past controversy about performing resummations in momentum space. Claims that

1. exponentiation is incomplete
2. momentum conservation is violated
3. there are large ambiguities, not related to Landau pole singularities.

Catani, Mangano, Nason, Trentadue '96

- 3. are not present in our formalism. Not sure what 1. and 2. mean.

Integral over structure function at LL

$$\mathcal{F}_2^{\rm ns}(x,Q^2) = \int_{1-x}^1 dy\, F_2^{\rm ns}(y,Q^2)$$

- LL, expand exponent in $a = \Gamma_0 \frac{\alpha_s(Q)}{8\pi}$

$$\mathcal{F}_2^{\rm ns}(x,Q^2) = \int_{1-x}^1 dy \sum_q e_q^2\, y\, \phi_q^{\rm ns}(y,Q)\, \exp\left[-a \ln^2\frac{\mu_i^2}{\mu_h^2} + 2a\ln\frac{\mu_i^2}{\mu_f^2}\,\ln(1-y)\right]$$

- With scale choice $\mu_f = \mu_h = Q,\ \mu_i \approx Q\sqrt{1-y}$

$$= \int_{1-x}^1 dy \sum_q e_q^2\, y\, \phi_q^{\rm ns}(y,\mu_f)\, \exp\left[a\ln^2(1-y)\right]$$

Nonintegrable singularity!

- Choose scales after integration!

Summary

- Traditionally, resummation for hard processes is performed in moment space.
  - Landau poles (in Sudakov exponent and Mellin inversion)
  - Mellin inversion only numerically
- Solving RG equations in SCET, we have obtained resummed expressions directly in momentum space.
  - Clear scale separation. No Landau pole ambiguities.
  - Simple analytic expressions.
  - Trivial connection with fixed order expressions.
- Same technology should be applicable to many other processes.
  - Threshold resummation for DY and Higgs production under way.